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THE FANTOPOLOGIST

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EDITORIAL

In presenting the "Fantopologist" we hope to bring to the attention of Fandom the latent and neglected possibilities of topology, or non-metric geometry, for stimulating creative thinking along literary and philosophical lines. We shall attempt the impossible of providing froth for the former and meat for the latter in quantities which we sincerely hope will satiate the appetites of both.

We invite articles, fiction, and poetry particularly devoted to the development of ideas new to science fiction. We believe there are several; topology simply happens to be one we know about.

-- H. T. McAdams
Editor

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THE INVISIBLE MARAUDER

By H. T. McAdams

Great had been the effort, but now even greater were the results. After ten years of untiring study and persistent improvement he had transcended the realm of human sight and experience.

He was a psychical investigator, first of all. Influenced by the Indian cult of theosophy, he had found that man real and unreal is composed of constituents of seven planes of existence. He had worked through these planes one by one -- the Physical, the Astral, the Mental, and the Buddhic -- and had now attained the fifth, the realm of Nirvana. His physical being seemed far, far behind.

This was a strange world, desolate and forsaken. But beautiful -- extremely beautiful. A dazzling effusion of living iridescence encircled his entire being and pervaded his very soul. In the midst of such beauty with physical eyes he certainly would have gone mad. But to Nirvanic eyes the sight was highly commendable -- beauty to its full capacity and no more. To such super-etheric senses as were his, even the scintillating opalescence of the astral plane is dull and disgusting, and sights in which the physical body takes special delight seem muddy and loathesome.

Days, or rather years, passed, either forward or backward, he knew not which -- Eternity knows no time. But he knew he loved this wonderful world, though desert, heavenly desert, it was. He loved it for its beauty, for its ecstatic loneliness, like the blue haze of an Indian Summer day, for the emotional strain induced by wandering listlessly through its sea of loveliness and color. For Beauty is to the emotions, what Knowledge is to the Intellect; in large quantities both are difficult, but satisfying, and yet not full satisfaction.

Driven only by an insane desire, as all theosophists are driven, as they believe all things are driven, he seemed at last to reach an impassable barrier, a point beyond which he could not go. An unseen force seemed to swerve him lightly aside and to direct his footsteps elsewhere. But one who has been fed upon success finds defeat nauseating. So with renewed vigor and perseverance his superhuman strength found for him a new world.

The abode of spirits! He knew the place. He could not describe it, he could not name it -- and yet he knew that he had been here before. It was nothing -- nothing but an eternal place. It was not real -- he knew it was not real. But it was here, and here was he, and here were the spirits of all men. Here were their fears, their pains, their sorrows, their hopes, their longings. He walked Aneas-like among them, the real among the unreal. Their filmy translucence taunted him, and he grew to abhor them, individually and severally, for they reminded him of a life that he had long since left behind. "Why," he asked himself, "why must Beauty lie so close to Ugliness?"

Time passed, loathing changed to hatred, and hatred bred revenge. He forgot the development he had been forced to make in order to reach this realm of nature. He forgot his years of spiritual study, his tiresome hours of mesmeric and necromantic exercise -- forgot that he must banish all passion if he was to train his clairvoyance sufficiently to be used. He remembered only one thing -- beauty. Here he found none. The implication was maddening!

Perhaps the intellect will always be ruled by the emotions. Here, where time both stayed and fled, it ruled. Finding that the objects of his hatred yielded supplely to his influence, he was suddenly seized by a thirst for manifested existence, an uncontrollable desire for revenge. He incited wars, crumbled empires, slew, tortured, and destroyed. All in all he played complete havoc on earth, and caused alone that train of events which people in their ignorance call history. He was, and still remains, an invisible marauder.

THE END

TOPOLOGY IN LITERATURE AND LIFE

"Of shoes and ships and sealing wax,
Of cabbages and kings."

Lewis Carroll's walrus undoubtedly was an adroit politician, and, as Jurgen would say, " monstrous clever fellow." In addition, however, there is good reason to believe that he was also, among other things, something of a topologist. For the topologist, like Carroll's walrus, is quite at home among various and sundry items such as knots, inner tubes, and pretzels.

Topology may perhaps best be defined as non-metric geometry. That is to say, it is not concerned with distances between points, sizes of angles, or any of the other measureable properties of geometric figures. To the uninitiated this definition will perhaps do little more than to evoke the exclamation, "Why, that is no geometry at all!" Let us consider, then, the following parable:

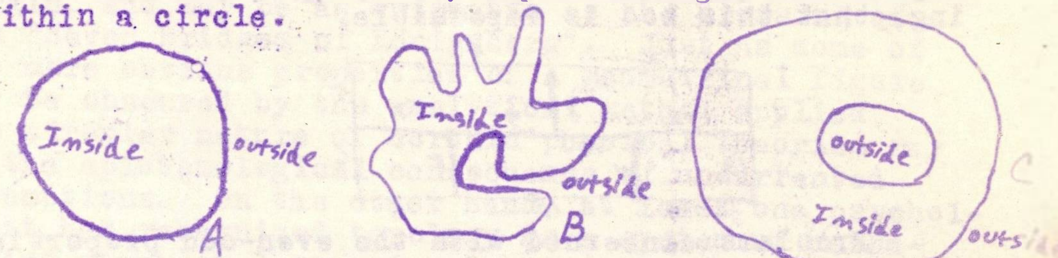
> In his cell of lithnon, Zeg Rah, imprisoned by the Lunar Spansi men, longed for his far-away home of Glyth. His heart cried out for the opalescent sheen of the bwa-tung trees in the dawnlight, for the liquid sound of the tekki birds as they glide gracefully along the surface of the sea and utter their plaintive cries, for the hush that falls when the great red ball of Glyth sinks below the horizon and leaves the Glythian planet to the scarcely perceptible music of a million vol-hush frogs, who seem loath to disturb the peace that is known only to peace-loving men. Most of all Zeg Rah longed for the languorous caresses of his beloved Cali Deh, for what were the bwa-tung trees, the tekki birds, or the tranquil songs of the vol-hush without love and the dreams of love? Regular telepathic communication with Cali Deh served only to sharpen to a stabbing hurt what otherwise would have been only a dull despair.

Then one day the Spansi Men with their clammy tentacles fastened a strange machine to the side of his cell, adjusted an interminable array of servo-mechanical controls, and disappeared forever. Every day, as at a pre-arranged time, Zeg Rah's cell would expand to twice its size, swallowing in turn after a time the moon itself, the entire solar system, and the myriad stars and island universes one by one. For a while Zeg Rah wondered whence came the fecund energy and the matter which went into the expanding lithnon cell, but before long became too enamoured with its very enormity to be much concerned with its method of manufacture. Each day, with his sub-atomic flight drive, which the Spansi had failed to remove from the inner pocket of his tokka jacket, Zeg Rah would explore his new accession. And each night, when he knew that the triple moons of Glyth would be shedding their soft amber light upon the bwa-tung trees, he would telepathically promise Cali Deh that soon, now, he would be home. And Cali Deh would respond with that sacred promise of undying love which is born only of hopes, and fears, and illusions.

And then, at last, the lithnon cell encompassed great Glyth, and the Glythian planets, and the bwa-tung trees, and all the rest -- and Zeg Rah knew that the time had come. But though he searched the planet inch by inch, traced and retraced his crazy paths through the bwa-tung trees, and listened to the liquid cries of the tekki birds and the quiet songs of the vol-hush until they drove him mad, Zeg Rah could nowhere find his Cali Deh.

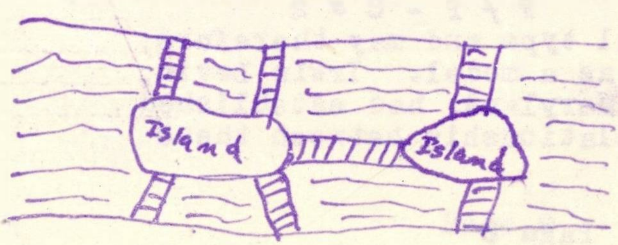
For all woman-life within the lithnon cell was dead. And Zeg Rah did not know the lithnon cell had once been woman-life.

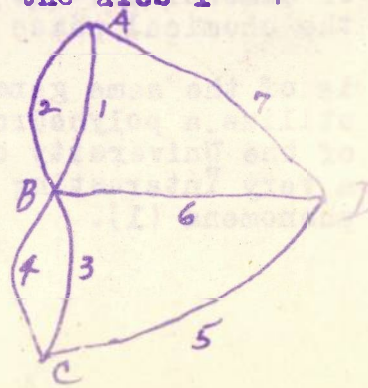
The question of inside and outside, then, is certainly a mathematical property independent of size, and one which is infinitely important to all of us, just as it was important to Zeg Bah. In fact, one of the most important of all topological theorems is that of Jordan, which states that any closed curve which does not cross itself divides the plane into an inside and an outside. This, in itself, is a simple enough statement perhaps, but requires some rather ingenious interpretation in cases such as C below, representing a circle within a circle.



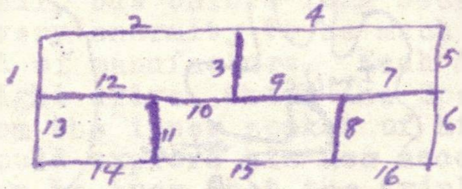
Here A and B are both Jordan curves, and are said to be topologically equivalent because one can be transformed into the other by stretching without tearing. This is not true of C, however, which is topologically of a different <u>genus</u> from A and B. In three dimensions a balloon is topologically different from an automobile inner tube, and in four dimensions -- well, let's stop there, shall we?

One of the celebrated problems of topology was concerned with "The Seven Bridges of Königsberg", the problem being to traverse all seven bridges in one continuous trip without retracing any of the path. In the conventionalized graph of the problem the vertices A and C represent the mainland, the vertices B and D the islands, and the arcs 1 - 7 inclusive the bridges.





Vertices are designated as even or odd accordingly as they are formed by the intersection of an even or an odd number of arcs. Euler showed that the number of trips required to traverse a graph is half the number of odd vertices which it contains. Therefore at least two trips are required in the case of the "Seven Bridges of Königsberg", for all four vertices are odd. An old problem of the same type is to cross all 16 segments of the following figure with a continuous line without crossing any segment twice. It can be shown, by Euler's reasoning, that this too is impossible.



Problems concerned with the even-odd properties of numbers, therefore, are of a topological nature. In the physical sciences the even-odd relationship has assumed significance in quantum mechanics, where Fermi-Dirac statistics are applicable to systems containing an odd number of fundamental particles (electrons, protons, neutrons), while Einstein-Bose statistics are applicable to systems containing an even number of such particles.

Another theorem attributed to Euler concerns the number of vertices, edges, and faces of a polyhedron. A cube, for example, has 8 vertices (V), 12 edges (E), and 6 faces (F). Thus it is easy to see that

$$V - E + F = 2$$

The relationship may be generalized to any number of dimensions. It is of singular interest that the chemical phase rule

$$F + P - C = 2$$

is of the same general type and may therefore utilize a polyhedron as a model. Irwin Levin, of the University of Maryland, has established a very interesting relationship between these phenomena (1).

It is to be seen, therefore, that metric properties by no means represent the sum total of geometry. Furthermore it is evident that if science continues to use space models to represent its theories -- a la Cartesian analytical geometry, or any other system of coordinates -- the topological rrlationships are not to be overlooked. Physicists are (2) beginning to realize that many of the discrepancies in their theories may be strictly topological in origin, and may be as enigmatic as the crossing of the "Seven Bridges of Königsberg". Just as some of the more obvious properties of a geometrical figure may be obscured by the analytical method applied, the circular nature of certain physical theories may be the epistemological consequence of unwarranted assumptions. On the other hand, at least one psychologist has recognized topology as a system of mathematics more or less uniquely adapted to cope with the problems of the mind, problems which have so far defied systematization by means of metric mathematics (3).

Topology has arrived at the frontier!

References:-

1. Journal of Chem. Ed. <u>23</u>, 183-185 (1946).
2. See (a) White - <u>Critique of Physics</u> and
   (b) Eddington - <u>The Philosophy of the Physical Sciences</u>.
3. Lewin, Kurt - <u>Principles of Topological Psychology</u>
4. A good general discussion of topology is given in Courant and Robbins. <u>What is Mathematics?</u>

FETTERS

Man dwells within small bounds
Of fear and self-conceit;
He avoids the open realms of thought
Even as he locks himself within four
walls,
Away from darkness.
He bounds his universe with mathematical
formulae,
And wonders, then, what lies beyond.

-- H. T. McAdams

DREAMS

Dreams are like the lapping
Of the waves upon the shore,
Bringing only bits of driftwood
From a far, far richer store.

-- H. T. McAdams

TOPOLOGY, NUMBER, AND WHAT WE CALL e

This article will attempt to show that the concept of dimension is largely a man-made, artificial one, particularly as to its limitation in number to three. It will not be easy reading for the novice, but for those who persevere it may present a new point of view. It will have accomplished its purpose if it succeeds in divorcing the idea of dimension from a finite number of coordinates, indeed from coordinates altogether. -- H.T.M.

In his "Introduction to Mathematical Philosophy" Bertrand Russell defines the number of a class as "the class of all those classes similar to it." By this, Russell means that ten apples, ten cows, and ten one-legged sailors each constitute a class of individuals unique from each other in some respects, but that there is a larger class which includes these classes, which in this case is the number ten. This definition, when carried to its logical conclusion, implies that there must somewhere be a given class of reference. The most abstract of all classes, and hence the one most suited for this purpose, is a point set, since it has no properties other than its cardinal number. In other words, each of the above ten individuals could have been put into one-to-one correspondence with a set of ten points, and this point set, considered ideally, is devoid of all sensible properties.

So far number has been considered only in its discrete or discontinuous aspects. When we consider a continuum, the concept of extension arises, and with it the concept of dimension. Any two points of the set define a linear continuum, which by definition, and by definition only, is said to be one-dimensional. This continuum, erstwhile a straight line in Euclidean space, may be more conveniently referred to as a 1-dimensional cell, or simply a 1-cell. A third point, chosen at random, _might_ be collinear with the other two and _might_ be contained in the 1-cell determined by them. However, it is not _necessary_ for this to be th case. The possibility of the existence of at least one point not contained in the given one-dimensional continuum, therefore, implies the existence of a space of higher order which we say is two-dimensional. Similarly, p points define a space having, at most, p-1 dimensions. Therefore a single point is zero-dimensional and the number of dimensions represented by an empty set (that is, no points) is -1, since in this case $p = 0$.

Space of n dimensions may be divided into an inside and an outside by space of n-1 dimensions. For example, a plane of two dimensions is divided into an inside and an outside by a Jordan curve, which is a closed line and hence one-dimensional. In the general case, the inside constitutes an n-dimensional cell, while the boundary comprises cells of n-1 dimensions. Each of these (n-1)-cells is bounded by (n-2)-cells, and so on until a point set of zero dimensions is reached. This point set is bounded by a cell of (-1) dimensions, which has no boundary. This is a consequence of the fact that in Euclidean space the minimum number of (n-1)-cells required to bound an n-cell is $n+1$. By the theory of combinations it may be shown that a given number of points p will establish at most:

1	(-1)-cell
p	0 -cells
$p(p-1)/2!$	1 -cells
$p(p-1)(p-2)/3!$	2 -cells And:
$p(p-1)(p-2) \ldots 1/p! = 1$	(p-1)-cell.

Therefore the maximum number of cells of -1, 0, 1, 2, ... (p-1) dimensions defined by p points is given by successive terms in the expansion of the binomial

$$(a - b)^p$$

when $a - b = 1$. For example, when $p = 3$,

$$(1-1)^3 = 1 - 3 + 3 - 1$$

and the interpretation is that three points determine one (-1)-cell, three 0-cells, three 1-cells, and one 2-cell.

It is to be observed in passing that the above expansion, by virtue of its numerical properties and the alteration of signs, constitutes a generalization of the Euler formula $V + F - E = 2$, relating the number of vertices, faces, and edges of a polyhedron. The real point of significance for our purpose here, however, is the fact that the function $(a - b)^p$ has been interpreted topologically by means of its expansion as a Taylor series. The inference is that Taylor's series, or in special cases, Maclaurin's se series, in expanding a function in terms of successive differentials, actually resolves that function into the several dimensional components of which it is composed. Certainly the differential coefficient of $f(x)$ is of one less dimension than the function, a fact well illustrated by the reduction in the power of the variable in

$$d\, x^n = n\, x^{n-1}\, dx.$$

The numerical coefficient n, being dimensionless, must refer to the properties of the point set coordinated to the function and is as significant an aspect of the dimensional transformation as is the exponent of the variable. An apparent anomaly is presented by

$$d\, e^x = e^x\, dx$$

in that the differential coefficient has apparently the same dimensions as the function itself. This simply implies, however, that the function represents an infinite number of dimensions, an implication supported by the fact that the expansion of e^x as a Maclaurin series has an infinite number of terms. In attaching significance to the individual terms of such an expansion, recourse must be had to an indirect approach inasmuch as in infinite number of points is involved in the representative set.

It has previously been intimated that two alternatives exist for the interpretation of number. One is the purely abstract, or topological, interpretation by which discontinuous entities are placed in one-to-one correspondence with the points of a set. The other is the quasi-concrete, or metric, interpretation by which an entity having the properties of a continuum of a given number of dimensions is expressed in terms of units of that continuum. The mensuration of n-dimensional space is conventionally effected in terms of orthogonal units having unit linear extension along each of its n perpendicular axes. Consequently the unit of content for 2-space is a square, that for 3-space a cube, that for 4-space an octahedroid, and so on. It is pointed out, however, that the square, cube and octahedroid do not represent the minimum configuration of points required for the definition of cells of two, three, and four dimensions respectively; the corresponding topological entities of maximum simplicity are the triangle, the tetrahedron, and the pentahedroid. By virtue of the fact that a line-segment is actually the simplest unit of one-dimensional space, the latter system of units possesses a topological continuity not accessible to the orthogonal system.

The content of a triangle, in square units, is $x_1x_2/2$, that of a tetrahedron, in cubic units, is $x_1x_2x_3/2\cdot3$, and that of the general n-cell established by $n + 1$ points is

$$(x_1x_2x_3 \ldots x_n)/n!$$

where the various x's refer to linear extension along the several axes. Now, if the linear extension along each of these axes is the same, the content of the general cell is $x^n/n!$, which is recognized as the coefficient of the general term in Maclaurin's series. More specifically, the content, in orthogonal units, of these "natural" unit cells is given by successive terms in the expansion

$$e^x = 1 + x + x^2/2! + x^3/3! + \ldots$$

where the nth term of the series refers to the content of a unit cell of n - 1 dimensions. When x is unity,

$$e^x = e = 1 + 1 + 1/2! + 1/3! + \ldots$$
$$= 2.71828$$

and e is arrived at as a logical consequence of the mensuration of an infinitely-dimensioned manifold in terms of orthogonal units.

It is known that e (2.71828) and pi (3.14159) constitute two of the most important of all numerical constants; it is likewise known that there are similar constants of primary importance in physical science -- namely, c, the velocity of light; h, Planck's constant, and a, the fine-structure constant. Some physicists, e.g. Eddington and White, believe that these constants, or at least the dimensionless ones, are a consequence of the logical process applied to the problems in which these constants arise. In view of the above discussion of e, they may be right.

IN TIMES TO COME

"The Pantopologist" is tentatively scheduled for bi-monthly publication. Your interest in the magazine, however, will largely determine its destiny. If and when the circulation will permit, we hope to graduate from gelatin, which we admit gives very poor duplication. Also as the publication progresses, we hope to recede more and more into the background in deference to material from the readers. Send us your ideas, be they serious or trivial; we intend to run the gamut just as we have done in this first issue. We plan to more or less follow the tradition of one or two stories, a popular article, a serious article, and one or more pages of poetry. In addition we expect to introduce appropriate book reviews and readers comments from time to time.

In the next issue we expect to discuss, among other things, topological group spaces and abstract metrics, with some hints as to their introduction into science fiction. We would also like to publish a historical survey of topology in science-fiction, if we can coax someone into writing it.

-- Mac

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